

In a nutshell: The Hooke-Jeeves method

Given a continuous real-valued function f of a vector variable with one initial approximation of a minimum \mathbf{u}_0 , the Hooke-Jeeves method steps towards a minimum by using the canonical unit vectors without relying on the ability to differentiate the function.

We will assume the dimension of the vector variable is n and the canonical vectors are $\mathbf{e}_1, \dots, \mathbf{e}_n$.

Parameters:

$\varepsilon_{\text{step}}$	The maximum error in the value of the minimum cannot exceed this value.
ε_{abs}	The difference in the value of the function after successive steps cannot exceed this value.
h	An initial step size.
N	The maximum number of iterations.

1. Let $k \leftarrow 0$.
2. If $k > N$, we have iterated N times, so stop and return signalling a failure to converge.
3. Let $\Delta \mathbf{u}_k \leftarrow \mathbf{0}$ and letting j take the values from 1 to n do the following:
 - a. If $f(\mathbf{u}_k + \Delta \mathbf{u}_k + h\mathbf{e}_j) < f(\mathbf{u}_k + \Delta \mathbf{u}_k), f(\mathbf{u}_k + \Delta \mathbf{u}_k - h\mathbf{e}_j)$, set $\Delta \mathbf{u}_k \leftarrow \Delta \mathbf{u}_k + h\mathbf{e}_j$,
 - b. otherwise, if $f(\mathbf{u}_k + \Delta \mathbf{u}_k - h\mathbf{e}_j) < f(\mathbf{u}_k + \Delta \mathbf{u}_k)$, set $\Delta \mathbf{u}_k \leftarrow \Delta \mathbf{u}_k - h\mathbf{e}_j$,
4. If $\Delta \mathbf{u}_k = \mathbf{0}$, we are done for this step, increment k and divide h by 2: $h \leftarrow h \div 2$, and return to Step 2.
5. Let $\mathbf{u}_{k+1} \leftarrow \mathbf{u}_k + \Delta \mathbf{u}_k$,
 - a. If $f(\mathbf{u}_{k+1} + \Delta \mathbf{u}_k) < f(\mathbf{u}_{k+1})$, set $\mathbf{u}_{k+1} \leftarrow \mathbf{u}_{k+1} + \Delta \mathbf{u}_k$ and return to this Step 5a.
6. If $\|\mathbf{u}_{k+1} - \mathbf{u}_k\|_2 < \varepsilon_{\text{step}}$ and $|f(\mathbf{u}_{k+1}) - f(\mathbf{u}_k)| < \varepsilon_{\text{abs}}$, return \mathbf{x}_{k+1} .
7. Return to Step 2.

Acknowledgement: Jakob Koblinsky noted I was referring to $\Delta \mathbf{x}_k$ and not $\Delta \mathbf{u}_k$ in Step 5. This has been corrected.